Percentiles and Wheelchair Spatial Modeling

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Preface

This essay has evolved out of research by Hunarch Consulting for the Australian Commonwealth Government into the spatial requirements of wheelchair and scooter use in buildings and facilities. It seeks to explain the difficulties caused by the expression of the research goal in terms of percentiles, and seeks to overcome common misconceptions about percentiles in anthropometrics and ergonomics.

Percentiles

A percentile is used to identify an entity within a group that is the largest of an equivalent percentage of that group, but smaller than the other entities. Hence, an entity at the 80th percentile is the largest of 80% of entities, but smaller than the remaining 20%. The entity at the nth percentile may be regarded as representative of n% of entities that are the same size as, or smaller than the entity.

One of the reasons we might want to identify an nth percentile entity is to determine the size of, say, a space that will accommodate that entity, to ensure that the space will therefore also accommodate all other group entitles this size and smaller. In other words, we wish to use the representative entity as a model in spatial determination.

Percentiles and parameters

A percentile value corresponds with an equivalent percentage inclusion of an entity only if the entity is defined by a single parameter or, if it is defined by more than one parameter, if there is perfect correlation between them, in other words, where an increase in the magnitude of one parameter is matched by an increase in magnitude of another. In such instances, an entity at the nth percentile will always be representative of n% of entities. By contrast, correspondence between the percentile value and percentage inclusion does not occur for disparate multivariate entities, i.e. entities that are defined by more than one parameter, and where one of them increases in magnitude as another decreases, or where there is less than perfect correlation between them.

This is easily illustrated. In Figure 1 are three rectangles, A, B and C depicting simplified outlines of fictitious wheelchairs[1] having lengths varying between 3 and 5 units, and widths varying between 1 and 3 units. We want to know the wheelchair that, in terms of length, is representative of two of the three wheelchairs, i.e. 67% of the wheelchairs, or in other words is at the

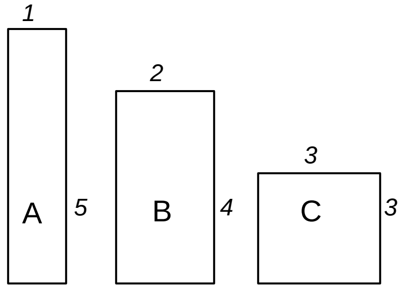

Figure 1

[1] The proportions of the "wheelchairs" illustrated throughout this article have been distorted for greater graphic clarity and to avoid the need for dimensions.

67^{th} percentile for length. The representative is wheelchair B. Similarly the wheelchair at the 67^{th} percentile for width, or in other words, the wheelchair that represents 67% is, again, wheelchair B.

However, let's say we wanted to know the wheelchair that, in terms of length *and* width is representative of 67% of wheelchairs. Again, the answer is B. However, this time we find that wheelchair B is no longer representative of 67%; instead, it is representative of only 33% of wheelchairs. This is because, whilst there is a perfect correlation between the wheelchair lengths and widths, it is a negative or inverse one, i.e. as they decrease in length they increase in width.

Let's consider a third parameter, say height, again in a very simplified way. Our rectangles now become cubes varying in height from 1 to 3 units, as shown in the aerial views in Figure 2. We can see that C, not B, represents 67% of the wheelchairs in terms of height, or in other words, is at the 67th percentile for height. But, which wheelchair is representative of 67% of the wheelchairs for length and width *and* height? We now have a problem. Because there is little correlation between depth and either of the other two parameters (there is no consistent increment or decrement in depth with increment or decrement in height or width), there is no representative wheelchair at the 67^{th} percentile for the three parameters.

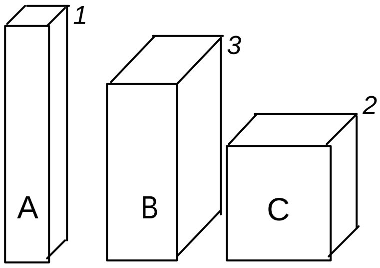

Figure 2

Now let's consider four rectangular wheelchair outlines that, like the above example, in comparison with each other decrease in length as they increase in width. The task this time is to identify the representative of 75% of wheelchairs. We can see that B is the representative for length, and that this time the representative for width is C. However, for length *and* width together there is no representative[2].

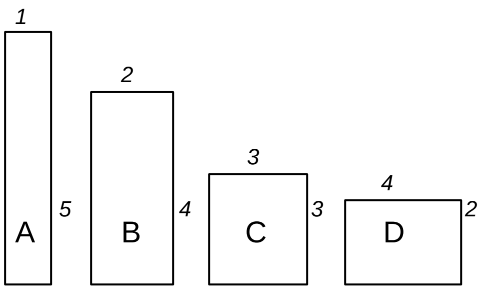

Figure 3

We can see from these examples that, for two or more disparate entities, there is no simple correspondence between a percentile and an equivalent percentage inclusion.

Percentiles, percentage inclusion and hypothetical models

If we accept that the purpose of identifying an entity at a percentile is to designate an equivalent percentage inclusion, we can accept that it is the achievement of the percentage inclusion that is the primary goal. For disparate multivariate entities for which there is no percentile representative, a means of overcoming this problem and of achieving n% inclusion is to create an imaginary representative, or hypothetical model for which one or more of its parameters are at a percentile value greater than the n.

[2] Such a dramatic effect would be less likely if the group were larger.

Using the three wheelchair example, let's assume that we wanted to determine a parking bay having the least length and width that will accommodate 67% of our wheelchairs. We know that there is no representative plan outline. However, we can achieve our 67% inclusion by creating an hypothetical model, or footprint, that is as long as B and as wide as C, or that is as wide as B and as long as A. The A/B and B/C models, shown in Figure 4, will each represent 67% of actual wheelchairs. It can be seen that the 67% inclusion now corresponds with one parameter that is at the 100th percentile: for the B/C model the width is at the 100th percentile, and for A/B the length is at the 100th percentile.

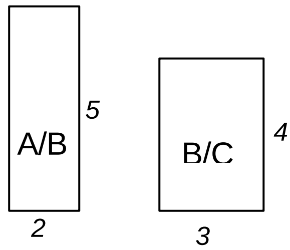

Figure 4

If there were more entities in the group, it is possible to increase the percentile value of each parameter by the same amount to achieve the desired percentage inclusion. For example, Figure 5 shows six rectangular wheelchair outlines. The top row shows the wheelchairs arranged in increasing width, and the bottom row shows them arranged in decreasing height (the fact that some of the wheelchairs are in different order in the two rows is indicative that the wheelchair lengths and widths are not perfectly correlated). We can see that C is at the 67th percentile for width and that B is at the 67th percentile for length. Again, there is no single representative for both length and width and so we seek to formulate an hypothetical model comprised of the length of B and the width of C. However, the only wheelchairs this size or smaller are B and C, therefore representing only 33%. To overcome this, we increase the percentile value to 75. Consequently, we find that E is at the 75th percentile for width and D is at the 75th percentile for length. The composite model therefore has the length of D and the width of E. This model does not represent 75% of wheelchairs, but it does represent 67%, which is our target.

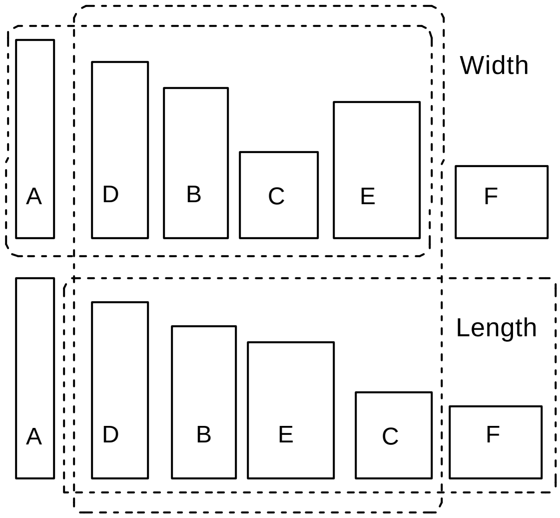

Figure 5

We can see from these examples that because there is no simple correspondence between the percentile and percentage inclusion, the key to achieving out percentage inclusion is to increase the percentile value.

Task definition, key parameters and models

Reverting to our earlier 3-wheelchair example, for which there were two possible hypothetical models, A/B and B/C as shown in Figure 4, another issue arises: which model is the correct one? The answer can be found in the purpose of the modeling. For example, let's say we needed to know the smallest parking bay, not only in terms of length and width, but also in terms of area, that would accommodate each of 67% of wheelchairs. We can see that the B/C model has a length of 4 and width of 3, hence an area of 12, whereas the A/B model has a length of 5 and a width of 2, hence an area of 10. The A/B model is therefore the one needed to determine the least-area, least length, least width parking bay.

An alternative approach could be to calculate the area of the wheelchairs, identify the 67th percentile and use it as the model to determine the parking bay size. However, another problem then emerges. We can see in Figure 1 that wheelchair B with an area of 8 is the representative wheelchair. But knowing the area does not enable us to determine the parking bay proportion: for example it could be 2 x 4, in which case only 33% of wheelchairs would be accommodated, or it could be 0.5 x 16, or 3 x 2.7, in which case no wheelchairs would be accommodated. Clearly, the model to be used must be defined by parameters relevant to and sufficient for the modeling task, i.e. be key parameters. In the case of the parking bay, if a least area bay with a least width or length were required, the relevant parameters would be either area and length, or area and width.

The assumption so far is that the parking bay will be rectangular. However, the least-area bay that will accommodate each of 67% of wheelchairs is not a rectangle but a shape corresponding with that of the model. But, as shown in Figure 6, the way in which the rectangles are superimposed, whether symmetrically or asymmetrically around a common vertical or horizontal axis (for example) will result in differently shaped models. After calculation, the least-area shapes are found to still be an A/B model, but of these which is the correct one? Again, this can only be resolved in terms of the modeling task. However, there are countless ways in which the outlines could be combined, all with different resulting shapes. Hence, for a least-area solution we would need to firstly define the shape of the space, by determining the alignment point of the outlines, in order to define the shape of the model. This could be tedious. In any case, relying upon task identification in order to formulate an hypothetical model, for spatial modeling purposes may be unnecessary.

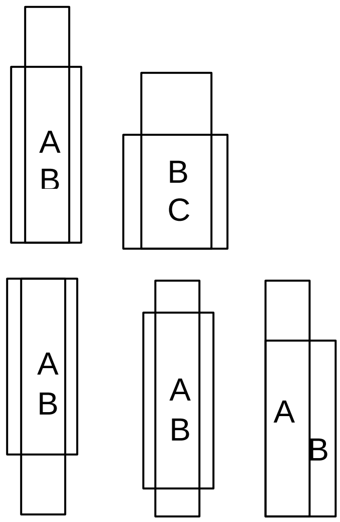

Figure 6

Multiple models

For our wheelchairs, it may be more convenient to use multiple models, with each model being an actual representative corresponding with a key parameter, than to use a single hypothetical model. For example, in all of the various parking bay tasks we can achieve our 67% inclusion by using two actual wheelchairs as models: one for the length parameter and one for width.

This can be further illustrated in relation to space for wheelchair rotation. Our task this time, with a different group of three wheelchairs, is to identify the model that we can use to determine the least-sized circle within which each of 67% of the wheelchairs can rotate 360º. There is only one key parameter: the largest of any diagonal line from a point midway on the drive wheel axis to the outer corners, hence only one parameter required to define the model

We can see from the size of the circles in Figure 7 that the 67th percentile representative is wheelchair A. Had we have adopted wheelchair B as the representative because it is at the 67th percentile for length and width, our result would have been incorrect.

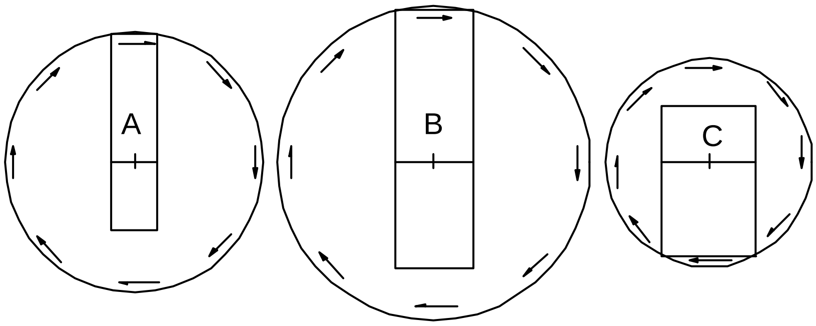

Figure 7

However, what if we wished to identify a model that would enable us to determine the smallest space in which an 180º rotation could be performed? As shown in Figure 8, for these rotations there are two radii that are relevant: the radius to the front and the radius to the rear.

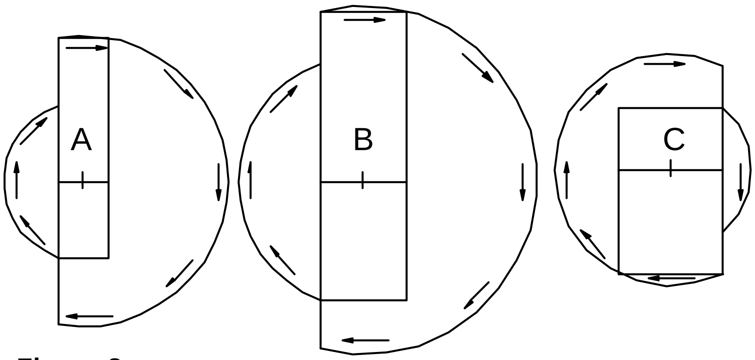

Figure 8

We can see that A is again the representative for the forward radius, and also that C is the representative for the rearward radius. But, similar to the parking bay examples, we see that there is no representative for both radii corresponding with the 67th percentile. Consequently, once again we could create a single hypothetical model. Because we are dealing with wheelchair rotation, the wheelchairs need to be aligned at the mid points of their drive wheel axes, and therefore the model would be as shown within the turning circle outlines shown in Figure 9.

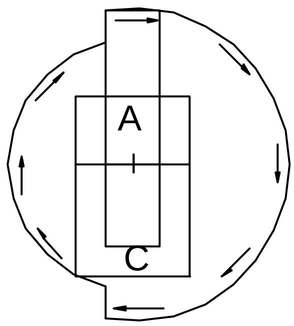

Figure 9

Whereas previously it was suggested that developing a single hypothetical model could require tedious task determination and be unnecessary, in this example we now see that, more importantly, to do so will be incorrect because it will not yield the smallest space to accommodate each of 67% of wheelchairs. This is made clear in Figure 10. The rotation space in terms of area and diameter (from left to right) is less for A than it is for C, with the space for the A/C model being larger than either. However, Figure 11 shows that the rotation space for C almost fully accommodates the rotation space for A. In other words, the smallest space that

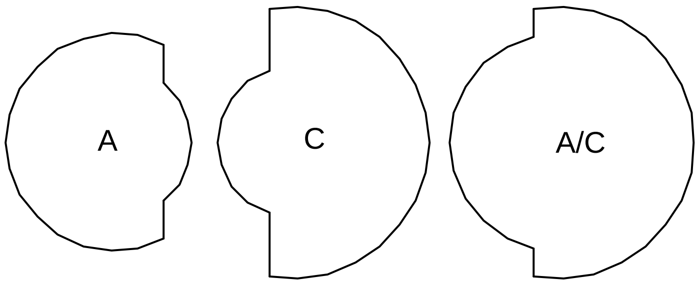

Figure 10

will accommodate each of 67% of wheelchairs is achieved by separately using each of the two representatives for the radii at the 67th percentile, wheelchairs A and C, not a single hypothetical model, A/C.

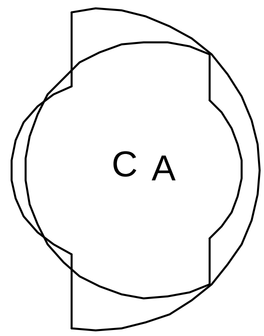

Figure 11

For this example, we can also see that the representative for length is again A and that for width it is also again B. Were we to have created a single hypothetical model out of A and B, with the mid-point of the drive wheel axes of A and B aligned, wheelchair B becomes, in effect, that model. However we can also see that the rotation area is larger than the A/C model. In other words, using length and width parameters to define the model by which to determine the rotation space gives an incorrect result, demonstrating once again the need to use parameters relevant to the task, in this case, radial parameters.

Multiple spatial tasks

What if we needed to know the model to define the least-sized parking bay as well as the least-sized 180º rotation space for the wheelchairs of the last example?

We know that there can be no single representative, and we have demonstrated that the use of a single hypothetical model is not as accurate as using separate actual models corresponding with the key parameters.

For multiple spatial tasks, there are two possible approaches: identify the models in terms of both tasks, or in terms of only one task.

For the first option, there would be four parameters: length, width, forward radius, and rearward radius. Hence, for the group size in our examples above, the parameters are represented by all wheelchairs and therefore constitute a 100% inclusion instead of the desired 67%. In fact it is not possible for these small group sizes to achieve the result we want. However, for larger group sizes, it is possible to achieve a target percentage inclusion using an inflated percentile value as previously illustrated, although a much larger group size than any illustrated here would probably be necessary for this.

For the second option, it can be reasoned that, because wheelchair travel is comprised of linear and rotational motion and that to arrive at a parking bay is impossible without traveling to it, parameters relevant to linear and rotational travel are the only key parameters. The key parameter in linear travel is width. Hence, for linear and rotational motion for our example there are three, not four key parameters: width, forward radius and rearward radius. Consequently there would be three models, not four.

On this basis, the task models for the second option may define a parking bay that is different to one defined by models specific to the parking bay. In fact, for the above example, this is the case.

Considering the parking bay by itself, the models are A and B, and by nesting them we achieve the required least-size parking bay, as shown in Figure 12. However,

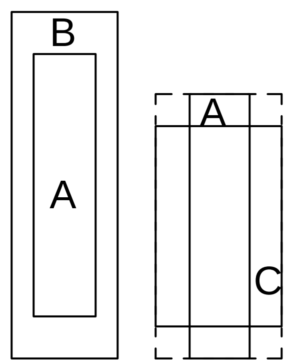

Figure 12

for the multiple task, A and C are the appropriate models because they represent the forward and rearward radii, and because the width of C accommodates the third key parameter, width (for which B is the representative of the group). However, in relation to width, we need only satisfy the parameter, not necessarily its representative model. If the width representative were wider than C, then the width model, B, would need to be satisfied. For this example therefore, the least sized area is found by nesting A and C, not A, B and C. Regardless of the shape of the parking bay, i.e. whether it is a rectangle, or a shape the same as the A/C outline, the area of an A/C parking bay is smaller than an A/B parking bay.

This issue of which models to use for a multiple task exercise could be, as before, regarded as one of task definition. But it is somewhat different than previously. It is therefore more appropriately characterised as a “policy” issue.

If the matter is one of policy then it could be regarded as extraneous to the discussion of percentiles. However, the issue does directly impact upon the matter of percentiles because fewer or different models will determine the extent to which the percentile value will need to be increased to achieve a target percentage inclusion.

Conclusion

It has been demonstrated that for disparate multivariate entities, such as occupied wheelchairs, percentile values typically do not correspond with an equivalent percentage inclusion. It has also been demonstrated that, whilst one way to overcome this problem in spatial modeling may be to formulate a single hypothetical model, a more accurate way is to use several models, each one being an actual representative of each key parameter. It has also been demonstrated that task and policy definitions may be necessary for identification of the key parameters.